Some Remarks on Solar eclipses: 
The curves "Earliest maximum eclipse" and "Latest maximum eclipse"

Robert Nufer

The original article [1] (in German) was published in ORION, the magazine of the Swiss Astronomical Society. Here I describe the key aspect, which may be of some interest to the SEML members. Therefore I rebuilt all the graphs with curve names in English. (Further, the orthographic views of Earth are replaced by screen shots of my eclipse inspecting program FInspektor).

Look at Figure 1: It is a stereographic projection of the total solar eclipse on 2006 March 29. The center line (red) begins at point C1 at sunrise and ends at sunset in point C2. Above and below the center line and the limits of total eclipse are the dashed curves (black) of equal magnitude (80%, 60%, 40% and 20%). The final limits of the partial eclipse are drawn as a continuous black line. The northern limit of the partial eclipse goes from PN1 to PN2 and southern limit of the partial eclipse goes from PS1 to PS2.

For this eclipse, the "rising" and "setting" curves form two oval regions (blue). The right one has the form of a distorted "8". According to Jean Meeus, this eclipse is of type IV [2]. For clarity, the curve names "Eclipse begins/ends at sunrise/set" are omitted. The lobes are intersected by the well known curves of "Maximum eclipse in the horizon". The "left" lobe is intersected by the curve "Maximum eclipse at sunrise", connecting PS1 with PN1, where the "right" lobe is intersected by the curve "Maximum eclipse at sunset", connecting PS2 with PN2.

---

![Total Solar Eclipse of 2006 March 29](image)

**Figure 1** The total solar eclipse of 2006 March 29 with curves "Maximum eclipse at a given time" (green).

- **Before 08:30**: Time interval 15 seconds
- **Between 08:30 - 08:35**: Time interval 1 minute
- **Between 08:35 - 11:45**: Time interval 5 minutes
- **Between 11:45 - 11:50**: Time interval 1 minute
- **After 11:50**: Time interval 15 seconds
Amazingly, only between C1 and PN1, the earliest maximum eclipse occurs with the sun in the horizon. In the southern half, the earliest maximum eclipse for each eclipse magnitude occurs along the magenta curve. For each eclipse magnitude there are for a short time, two points on the Earth surface having maximum eclipse at the same time. One of these points moves in the "wrong direction to the left" toward the sunrise. The other point moves over the earth as expected. Between C1 and PN1, the curves "Maximum eclipse at sunrise" and "Earliest maximum eclipse" coincide. The corresponding effect can be observed in the ending region of the solar eclipse, where the latest eclipse occurs not between C2 and PN2, but along the magenta curve. Between C2 and PS2, the curves "Maximum eclipse at sunset" and "Latest maximum eclipse" coincide.

Apparently, the progress "in time" is not as easy to understand as the progress "in geography". In fact, two observers at different places on Earth can see an eclipse maximum with same magnitude at the same time!

This "strange effect" was noticed by Jean Meeus when he calculated not only solar eclipses, but occultations of stars by the Moon, many years ago [3]. In his newest book, Mathematical Astronomy Morsels IV, Chapter 19 covers this "StEf" or "Meeus effect". [4]

The effect is shown in Figure 2. Between the red curves with time intervals of one minute, green curves with time intervals of fifteen seconds are drawn.

On the southern limit of the partial eclipse, the earliest maximum eclipse (with magnitude zero) occurs at 08:26:27 (PS0). Left and right of PS0 there are the 15 second intervals of 08:26:30 and 08:26:45 (green), followed by the integer minute 08:27 (red), and finally the time point 08:27:15 (green) as denoted by the arrows.

This last time point is the moment where an observer at PS1 "sees" a zero magnitude eclipse in the horizon at sunrise, but an observer at the other arrow "sees" a zero magnitude eclipse with the sun at an altitude of more than 15 degrees.

But PS0 is only the earliest maximum eclipse on the southern limit. The absolute earliest maximum eclipse occurs at PM0, i.e. 30 seconds earlier. The eclipse magnitude at this point is about 20%.

Principally, all the curves "Maximum eclipse at a given time" are closed curves on the Earth's surface, but for the most time of the eclipse they are limited by the Moon's shadow or the day-night limit with the Sun in the horizon.

Whether the closed curves "Maximum eclipse at a given time" occur or not depends on the geometrical and dynamical conditions of the eclipse. Figures 3 to 5 show three eclipses with increasing gamma.

If Earth would not rotate relative to the Sun's direction, maximum eclipse would occur along the red intersection of the penumbra perpendicular to the shadow path (Figures 3 to 5, Bottom).
Figure 3  Solar eclipse with $\gamma = -0.0041$. Top: Stereographic projection with the curves “Earliest maximum eclipse” and “Latest maximum eclipse” (magenta). Bottom: The penumbra near the end of the central eclipse.
Figure 4  Solar eclipse with $\gamma = +0.3306$. Top: Stereographic projection with the curves "Earliest maximum eclipse" and "Latest maximum eclipse" (magenta). Bottom: The penumbra near the end of the central eclipse.
Total Solar Eclipse of 2008 August 01

Figure 5  Solar eclipse with $\gamma = +0.8307$. Top: Stereographic projection with the curve "Latest maximum eclipse" (magenta). The curve "Earliest maximum eclipse" coincide with the curve "Maximum eclipse at sunrise". Bottom: The penumbra near the end of the central eclipse.
Calculation method

To get the curves "Earliest maximum eclipse" and "Latest maximum eclipse" the points for the curves of equal magnitude were calculated within 0.2 % magnitude steps at a resolution of 0.2 degrees in geographical longitude [5]. For each point the coordinates and the time were stored. At this stage, every iso-magnitude was represented by a cluster of points. Sorting these clusters in the time domain yielded the two curves. To draw the iso-magnitude curves from one end to the other, the clusters must be reorganized in a "logical geographical order".

The curves "Maximum eclipse at a given time" were also interpolated from the iso-magnitude curves. With this method, it is possible to get the closed curves, for example the red curve at 08:26 in Figure 2.

Of course, the curves "Earliest maximum eclipse" and "Latest maximum eclipse" as well as the occurrence of the "strange effect" have no relevant implications to "eclipse chasers". Nevertheless, they are phenomena with interesting side effects, which is a “bonus” for the truly amazing Solar eclipses.

References